EFFICIENT NON-UNIFORM FILTER-BANK EQUALIZER

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ABSTRACT

An efficient realization of a low delay filter-bank will be proposed, which can be regarded as a filter-bank used as equalizer with either fixed or time-varying spectral gain factors. The proposed filter-bank equalizer (FBE) results in a time-domain filter whose coefficients are adapted in the (short-term) spectral-domain. Perfect signal reconstruction is obtained for a broad class of spectral transforms, including the generalized discrete Fourier transform (GDFT), the Walsh and Hadamard transform, with less restrictions compared to a common analysis-synthesis filter-bank (AS FB). A non-uniform frequency resolution can be achieved by frequency warping based on an allpass transformation. In this case, the filter-bank equalizer can achieve near perfect signal reconstruction with lower effort than for an allpass transformed analysis-synthesis filter-bank. The filter-bank equalizer is especially useful for adaptive filtering requiring a low signal delay with coefficient adaptation based on frequency-domain algorithms.

1. INTRODUCTION

Many algorithms for digital signal processing are based on (short-term) spectral-domain filtering. A common approach to realize such algorithms is to employ a (DFT) *analysis-synthesis filter-bank (AS FB)* as illustrated in Fig. 1. It shows how the M complex modulated
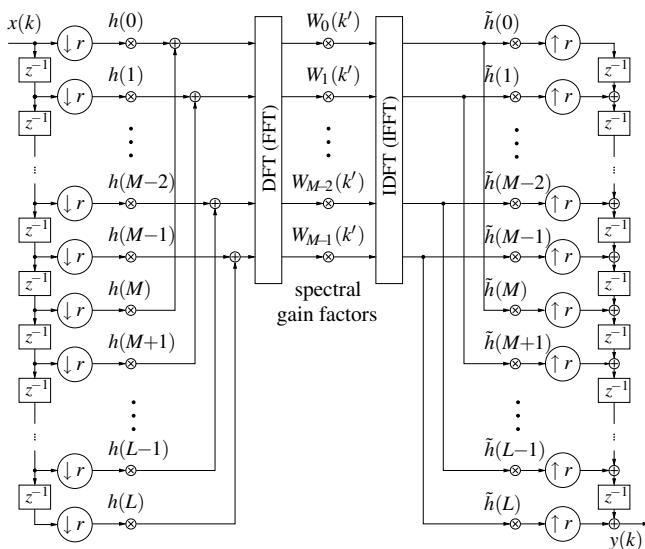

Figure 1: *DFT polyphase network analysis-synthesis filter-bank (PPN AS FB) for a prototype filter length of $L+1 = 2M$. An all-pass transformed filter-bank (APT FB) arises by replacing the delay elements with allpass filters.*

sub-band filters of a DFT AS FB with down-sampling can be efficiently realized by a polyphase network with the DFT calculated by the Fast Fourier Transform (FFT), e.g., [1]. The sample index k'

indicates that the spectral gain factors $W_i(k')$ are adjusted at a lower rate than the sampling rate, where $k' r = k$. The calculation of the spectral gain factors has not been drawn for the sake of clarity.

An AS FB with non-uniform frequency resolution can be obtained by an allpass transformation of the sub-band filters [2],[3]. The frequency resolution of such an allpass transformed analysis-synthesis filter-bank (APT AS FB) can be adapted to the Bark scale which is beneficial, e.g., for noise reduction [4].

A filter-bank (FB) with a significantly lower signal delay compared to the AS FB is shown in Fig. 2, hence termed as low delay filter-bank here. This filter-bank has a significantly higher algorith-
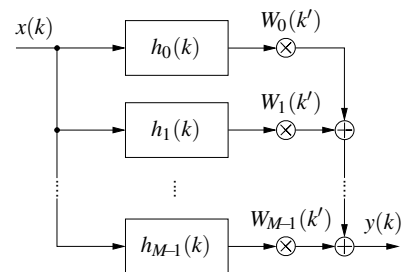

Figure 2: *Low delay filter-bank with time-varying gain factors.*

mic complexity, in terms of computational complexity and memory consumption, compared to the polyphase network analysis-synthesis filter-bank (PPN AS FB) with down-sampling of Fig. 1.

An efficient implementation of the uniform and allpass transformed DFT/ DCT low delay filter-bank, denoted as adaptive filter-bank equalizer (FBE), has been proposed in [5]. A similar allpass transformed DFT filter structure (for the case $L+1 = M$) has been proposed independently in [6] for the use as dynamic range compressor in hearing aids.

In this paper, the concept of the filter-bank equalizer will be extended and generalized. The generalized uniform FBE will be derived in Sec. 2, and conditions for perfect (signal) reconstruction will be established. The allpass transformed filter-bank equalizer (APT FBE) will be introduced in Sec. 3. It will be shown how to obtain an overall phase response for the FBE which is independent of the (time-varying) gain factors. These results will then be used to present an APT FBE with near linear phase and near perfect reconstruction, respectively. An efficient polyphase network realization of the FBE will be developed in Sec. 4. The algorithmic complexity and signal delay of the presented FBE will be compared with that of the AS FB in Sec. 5. A summary is given in Sec. 6.

2. UNIFORM FILTER-BANK EQUALIZER

The *low delay filter-bank* of Fig. 2 with M sub-bands will be regarded. The impulse response $h_i(n)$ of the i-th sub-band filter shall be given by a modulation of the prototype lowpass filter with impulse response $h(n)$ of length $L+1 \geq M$ according to

$$h_i(n) = \begin{cases} h(n)\, \Phi(i,n) & ;\ i = 0,1,\ldots,M-1;\ n = 0,1\ldots,L \\ 0 & ;\ \text{else} . \end{cases} \quad (1)$$

$\Phi(i,n)$ represents a general modulation sequence and can be regarded as the transformation kernel of the FB. The modulation sequence and the kind and length of the prototype filter determine the spectral selectivity of the FB. The relation

$$\Phi(i,n)\,\tilde{p}(m) = \Phi(i,n+mM) \; ; \; m \in \mathbf{Z} \qquad (2)$$

will be used in the sequel to express the periodicity of the transformation kernel in general. The sequence $\tilde{p}$ is defined but becomes dependent on n for non integer valued arguments.

An important transformation kernel is given by the generalized discrete Fourier transform (GDFT)

$$\Phi_{\mathrm{GDFT}}(i,n) = \exp\left\{-j\,\frac{2\pi}{M}\,(i-i_0)\,(n-n_0)\right\} \; ; \; i_0,n_0 \in \mathbf{R} \quad (3)$$

with the DFT obtained for $i_0 = n_0 = 0$. ($\mathbf{Z}$ denotes the set of all integer numbers, $\mathbf{R}$ all real numbers, and $\mathbf{C}$ all complex numbers.) This modulation sequence possesses the periodicity of Eq. (2) with

$$\tilde{p}(m) = \exp\left\{2\pi\, m\, i_0\right\} \; . \qquad (4)$$

The input-output relation for the FB of Fig. 2 is given in the z-domain for time-invariant spectral gain factors $W_i(k') = W_i$ by

$$Y(z) = \sum_{i=0}^{M-1} W_i\left(\sum_{n=0}^{L} X(z)\,z^{-n}\,h_i(n)\right) \; . \qquad (5)$$

The overall transfer function $F_0(z) = Y(z)/X(z)$ is obtained by inserting Eq. (1)

$$F_0(z) = \sum_{n=0}^{L} h(n) \underbrace{\sum_{i=0}^{M-1} W_i\,\Phi(i,n)}_{w_n \doteq T\{W_i\}}\,z^{-n} \qquad (6)$$

$$= \sum_{n=0}^{L} h(n)\,w_n\,z^{-n} \doteq \sum_{n=0}^{L} h_s(n)\,z^{-n} \; . \qquad (7)$$

The (time-domain) weighting factors w_n are evaluated by spectral transformation $T\{W_i\}$ of the gain factors. The weighting factors have a periodicity which follows form Eq. (2) and Eq. (6)

$$w_n\,\tilde{p}(m) = w_{n+mM} \; . \qquad (8)$$

For the GDFT of Eq. (3), $\tilde{p}(m)$ is given by Eq. (4) such that

$$w_n\,e^{j2\pi m i_0} = w_{n+mM} \; . \qquad (9)$$

The derived filter(-bank) structure will be termed as *generalized filter-bank equalizer (FBE)* since this concept applies for a broad class of modulation sequences (spectral transforms). The FBE is a single filter whose impulse response $h_s(n)$ is the product of the impulse response $h(n)$ of the prototype filter and the weighting factors w_n calculated in the (short-term) spectral-domain. However, the naming as 'filter-bank equalizer' should point out that this kind of time-domain filtering has been developed from the FB of Fig. 2 which can be regarded as a FB used as equalizer.

The low delay FB of Fig. 2 and the derived efficient implementation - the FBE - are exactly equivalent for fixed gain factors. The adaptive FBE, illustrated in Fig. 3, is obtained for time-varying spectral gain factors $W_i(k')$. The gain adaption can be done at decimated sample instants k' by means of a PPN analysis FB according to Fig. 1 (cf. [7]). The possible time dependency of coefficients by k' will be omitted in the following for the sake of simplicity.

General conditions for perfect signal reconstruction shall be established where $W_i = 1$. If the transformation kernel possesses the property

$$\sum_{i=0}^{M-1} \Phi(i,n) = \begin{cases} c \neq 0 & ; \; n = n_0 \\ 0 & ; \; n \neq n_0 \end{cases} ; \; n,n_0 \in \{0,1,\ldots,M-1\} \quad (10)$$
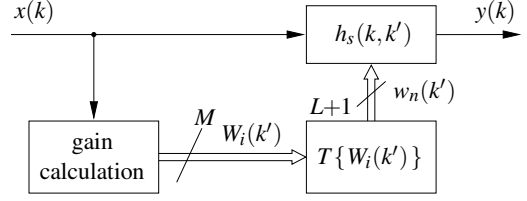

Figure 3: *Adaptive generalized filter-bank equalizer (FBE).*

(where mostly $c = M$) the overall transfer function of the FBE according to Eq. (6) can be expressed by

$$F_0(z) = c \cdot \sum_{n=0}^{L} h(n)\,\delta_M(n-n_0)\,\tilde{p}\left(\frac{n-n_0}{M}\right) z^{-n} \qquad (11)$$

due to Eq. (2). The expression $\delta_M(n)$ represents the comb-sequence

$$\delta_M(n) = \begin{cases} 1 & ; \; n = mM; \; m \in \mathbf{Z} \\ 0 & ; \; \text{else} . \end{cases} \qquad (12)$$

Employing a generalized M-th band filter with impulse response

$$h(n) = \begin{cases} (c\,\tilde{p}(m_c))^{-1} & ; \; n = n_0 + m_c\,M \doteq d_0; \; \tilde{p}(m_c) \neq 0 \\ 0 & ; \; n = n_0 + mM; \; m \in \mathbf{Z}\backslash\{m_c\} \\ \text{arbitrary} & ; \; \text{else} \end{cases} \quad (13)$$

as prototype lowpass filter yields the transfer function

$$F_0(z) = z^{-d_0} \; , \qquad (14)$$

that is, the uniform FBE ensures *perfect signal reconstruction* with a signal delay of d_0 samples.

Eq. (10) holds for a variety of spectral transforms, such as the GDFT, the Hadamard and the Walsh transform (e.g. [8]). In contrast, Eq. (10) is not valid for the Haar transform.

It can be concluded that perfect reconstruction can be obtained for the uniform FBE for a broad class of spectral transforms and less restrictions compared to the uniform AS FB. For time-invariant gain factors, the FBE is a linear, time-invariant (LTI) system, whereas the AS FB is a linear, periodically time-varying (LPTV) system.

3. NON-UNIFORM FILTER-BANK EQUALIZER

An approach to obtain a filter(-bank) with a non-uniform frequency resolution is to employ digital frequency warping by means of an allpass transformation [2],[3]. Thereby, the delay elements of the discrete (sub-band) filters are replaced by allpass filters

$$z^{-1} \; \rightarrow \; H_A(z) \; . \qquad (15)$$

A (causal) allpass filter of first order with frequency response

$$H_A(z = e^{j\Omega}) = \frac{1 - a^*\,e^{j\Omega}}{e^{j\Omega} - a} = e^{-j\varphi_a(\Omega)} \qquad (16)$$

$$|a| < 1 \; ; \; a = \alpha\,e^{j\gamma} \in \mathbf{C}$$

will be used for the allpass transformation here. The phase response of this allpass filter is given by

$$\varphi_a(\Omega) = -\Omega + 2\arctan\left(\frac{\sin\Omega - \alpha\sin\gamma}{\cos\Omega - \alpha\cos\gamma}\right) \; . \qquad (17)$$

The frequency response of the *allpass transformed filter-bank equalizer (APT FBE)* is derived from Eq. (7) by applying Eq. (15) and Eq. (16)

$$F_a(z = e^{j\Omega}) = \sum_{n=0}^{L} h(n)\,w_n\,e^{-jn\,\varphi_a(\Omega)} \; . \qquad (18)$$

Thus, the allpass transformation causes a frequency mapping[1] $\Omega \rightarrow \varphi_a(\Omega)$. This frequency mapping (warping) is solely determined by the allpass pole a according to Eq. (17). The uniform FBE with transfer function $F_0(z)$ is included as special case for $a=0$ since then $H_A(z)=1/z$.

An important scenario is given if the (possibly complex) impulse response $h_s(n)$ of Eq. (7) possesses the symmetry

$$h_s(n) = h_s^*(L-n) \;\; ; \;\; n = 0, 1, \ldots, L \; . \tag{19}$$

It can be shown that Eq. (18) can now be expressed by

$$F_a(e^{j\Omega}) = e^{-j\frac{L}{2}\varphi_a(\Omega)} \cdot \begin{cases} \sum\limits_{n=0}^{\frac{L}{2}} 2A(n,L,\Omega) - h_s\left(\frac{L}{2}\right) \;\; ; \;\; L \text{ even} \\ \sum\limits_{n=0}^{\frac{L-1}{2}} 2A(n,L,\Omega) \quad\quad\quad ; \;\; L \text{ odd} \end{cases} \tag{20}$$

noting that $h_s(L/2)$ is being real and with $A(n,L,\Omega)$ given by

$$A(n,L,\Omega) \doteq \mathrm{Re}\left\{h_s(n)\right\} \cdot \cos\left(\left[\frac{L}{2} - n\right]\varphi_a(\Omega)\right)$$
$$- \mathrm{Im}\left\{h_s(n)\right\} \cdot \sin\left(\left[\frac{L}{2} - n\right]\varphi_a(\Omega)\right) \; . \tag{21}$$

Similar results are obtained for an impulse response with symmetry $h_s(n) = -h_s^*(L-n)$, but such filters can not realize a lowpass.

The phase response for Eq. (20) can be regarded as independent of the (fixed or time-varying) filter coefficients $h_s(n)$ since the terms to the right of the curly brace are real and cause therefore only phase shifts by π. Thus, the uniform FBE ($a=0$) has then a generalized linear phase. For the APT AS FB, a 'fixed' overall phase response can not be obtained for time-varying spectral gain factors by imposing comparably simple symmetry restrictions as in Eq. (19).

The exact expression for the overall phase response of the APT FBE allows to employ a *fixed phase equalizer* to compensate its non-linear phase response and to achieve approximately a (generalized) linear phase. In this manner, an APT FBE with *near perfect signal reconstruction* can be obtained as well as Eq. (14) is now given by $F_a(e^{j\Omega}) = \exp\{-j\,d_0\,\varphi_a(\Omega)\}$ due to Eq. (15) and Eq. (16). A perfect phase equalization can not be achieved for finite group-delay and finite complexity: The needed 'ideal' phase equalizer with frequency response

$$P_{\mathrm{ideal}}(e^{j\Omega}) = e^{j\left(\frac{L}{2}\varphi_a(\Omega) - \tau_0\Omega\right)} \;\; ; \;\; 0 < \tau_0 < \infty \tag{22}$$

has a non-causal, infinite impulse response $p_{\mathrm{ideal}}(n)$ for $a \neq 0$. However, the overall phase response of the APT FBE can be 'linearized' by approximating the ideal phase equalizer of Eq. (22) by a causal allpass filter. A method to construct an allpass filter with prescribed phase response (which is optimal in a least-square sense) has been proposed in [9]. An FIR phase equalizer with filter degree L_{pe} can be obtained by approximating the ideal phase equalizer of Eq. (22) by truncation of its impulse response with, e.g., a rectangular window

$$p_{\mathrm{FIR}}(n) = \begin{cases} p_{\mathrm{ideal}}(n) \;\; ; \;\; n = 0, 1, \ldots, L_{\mathrm{pe}} \text{ and } \tau_0 = L_{\mathrm{pe}} \\ 0 \quad\quad\quad ; \text{ else} \; . \end{cases} \tag{23}$$

The truncation by a rectangular window provides the least-square approximation error. The FIR phase equalizer is no allpass and causes therefore amplitude distortions dependent on the degree L_{pe}.

The allpass transformation of an AS FB without down-sampling and perfect reconstruction yields the frequency response

$$F_a(e^{j\Omega}) = e^{-jL\,\varphi_a(\Omega)} \; . \tag{24}$$

[1]An allpass filter of degree N_a maps the unit-circle N_a-times onto its own and the function $\varphi_a(\Omega)$ is no longer injective.

If this relation is approximately fulfilled for sub-band filtering with down-sampling, a phase equalizer can be constructed in the same way as shown for the APT FBE [4].

Comparison of Eq. (24) with Eq. (20) reveals that the non-linear allpass phase $\varphi_a(\Omega)$ is now amplified by L instead of $L/2$, leading to a higher group-delay for the APT AS FB than for the APT FBE. Another consequence is that the phase equalizer for the APT FBE can have half the filter degree (complexity) than the phase equalizer for the corresponding APT AS FB for approximately the same phase distortions (reconstruction error). Employing the FIR phase equalizer of Eq. (23) leads to an overall group-delay which is approximately constant and equal to L_{pe}. Thus, the signal delay of the APT FBE with phase equalizer is about half of that for the corresponding APT AS FB with phase equalizer in this case.

Thereby, the choice of the filter degree for the phase equalizer is subject to the trade-off between phase distortions and, if so, amplitude distortions on one hand, and increased signal delay (average group-delay) and computational complexity on the other hand. For speech processing, for instance, an APT FB with near linear phase is usually not needed as the human ear is relatively insensitive towards phase distortions.

4. POLYPHASE NETWORK REALIZATION

An efficient polyphase network realization of the generalized FBE shall be developed. The (type 1) polyphase components of the prototype filter are given in the z-domain by (e.g. [1])

$$H_\lambda^{(M)}(z^M) = \sum_{m=-\infty}^{\infty} h(\lambda + mM)\,z^{-mM} \;\; ; \;\; 0 \leq \lambda \leq M-1 \; . \tag{25}$$

Therefore, the overall transfer function of the uniform FBE according to Eq. (7) can be expressed by Eq. (25) and Eq. (8)

$$F_0(z) = \sum_{\lambda=0}^{M-1} w_\lambda \sum_{m=0}^{\lceil\frac{L+1}{M}\rceil-1} \tilde{p}(m)\,h(\lambda + mM)\,z^{-(\lambda+mM)} \; . \tag{26}$$

(The operation $\lceil a \rceil$ provides the smallest integer which is equal or greater than a.) In many cases $\tilde{p}(m) = 1$ which yields

$$F_0(z) = \sum_{\lambda=0}^{M-1} w_\lambda H_\lambda^{(M)}(z^M)\,z^{-\lambda} \; . \tag{27}$$

The developed *uniform polyphase network filter-bank equalizer (PPN FBE)* is illustrated in Fig. 4. This implementation accounts,

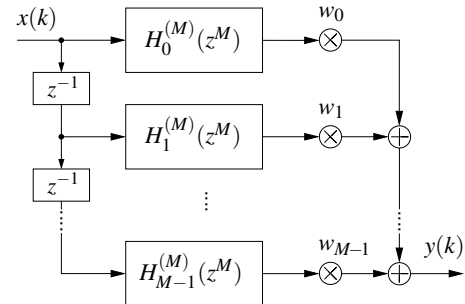Figure 4: *Uniform polyphase network filter-bank equalizer (PPN FBE) for the case of $\tilde{p}(m)=1$.*

e.g., for the Walsh and Hadamard FBE, as well as for the DFT FBE (cf. Sec. 2). The oddly stacked DFT FB is given for the GDFT of Eq. (3) with $i_0 = 1/2$. The PPN FBE has then the transfer function

$$F_0(z) = \sum_{\lambda=0}^{M-1} w_\lambda \cdot \left(H_\lambda^{(2M)}(z^{2M})\,z^{-\lambda} - H_{M+\lambda}^{(2M)}(z^{2M})\,z^{-(M+\lambda)}\right) \tag{28}$$

due to Eq. (26) and Eq. (9). It should be noted that the polyphase representation according to Eq. (27) and Eq. (28) have both the same algorithmic complexity. The polyphase representations for other periodicities of the weighting factors, i.e., other transformation kernels, are derived alike form Eq. (26). The *allpass transformed PPN FBE* for Eq. (27) is given by

$$F_a(z) = \sum_{\lambda=0}^{M-1} w_\lambda \cdot H_\lambda^{(M)} \left(H_A^{-M}(z) \right) \cdot H_A(z)^\lambda \qquad (29)$$

and accordingly for other PPN FBEs. The PPN FBE applies for an FIR and IIR prototype filter. The use of prototype filters with minimal phase is possible with less restrictions than for the AS FB.

5. COMPARISON OF FBE AND AS FB

5.1 Algorithmic Complexity

Table 1 contrasts the algorithmic complexity of the derived (APT) PPN FBE to that of the (APT) PPN AS FB of Fig. 1.

	spectral transform (2 real FFTs)	remaining FB operations	additional OPs for APT
polyphase network analysis-synthesis filter-bank (PPN AS FB)			
mult.	$2M/r \log_2 M$	$(2L+2+M)/r$	$4L$
add.	$3M/r \log_2 M$	$(L-M+1)/r+L$	$4L$
mem.	$2M$	$2L$	-
polyphase network filter-bank equalizer (PPN FBE)			
mult.	$2M/r \log_2 M$	$L+1+M$	$2L$
add.	$3M/r \log_2 M$	L	$2L$
mem.	$2M$	L	-

Table 1: *Algorithmic complexity in terms of required average number of real multiplications and real additions per sample, and number of delay elements (memory) for different realizations of a DFT FB. A real prototype filter and a real input sequence $x(k)$ are assumed. The last column contains the additional operations needed for the allpass transformation of the filter-bank.*

The used allpass filters has a real pole a and can then be realized by two (real) multiplier, two adders, and one delay element. The real DFT of size M - computed in-place by the radix-2 FFT algorithm - has been considered as transformation kernel. (The GDFT can be calculated by means of the FFT with similar complexity than for the DFT.) Thereby, the FFT of a real sequence of size M can be computed by means of a complex FFT of size $M/2$, i.e., with approximately half the algorithmic complexity than for a complex M-point FFT. The real FFT has to be computed twice after r sample instants for both filter-bank structures, assuming real spectral gain factors $W_i(k')$. For example, this is the case for many noise reduction algorithms based on spectral subtraction, cf. [4], [7]. For the AS FB the rate for the computation of the FFTs is governed by restrictions for signal reconstruction which is not the case for the FBE where the FFTs are only needed to update the weighting factors. The computational complexity for the calculation of the spectral gain factors is independent of the FB and has therefore not been considered here.

Table 1 reveals that the uniform PPN FBE requires less summations but more multiplications than the uniform PPN AS FB for most parameter configurations of L, M, and r. An important advantage of the FBE is the lower number of required delay elements (memory) compared to the corresponding AS FB with the same values for L, M, and r.

For speech enhancement with frequency warping, typical filter-bank parameters are, for example, $L+1 = 4M = 1024$ and $r = 64$ [4]. According to Table 1, the APT PPN AS FB then needs 4192 multiplications and 5223 summations per sample, as well as 2558 delay

elements. In contrast, the APT PPN FBE needs 4413 multiplications and 3165 summations per sample, and 1535 delay elements. Thereby, the complexity for the phase equalizer has not been considered. For example, the FIR phase equalizer according to Eq. (23) needs $L_{pe}+1$ multiplications, L_{pe} summations, and L_{pe} delay elements. As discussed in Sec. 2, the filter degree (i.e. computational complexity) of the phase equalizer for the APT FBE can be half of that for the APT AS FB for about the same reconstruction error. Hence, the APT FBE has a lower computational complexity than the corresponding APT AS FB for most configurations of the FB parameters (L, M, r).

5.2 Signal Delay and Application

As shown in Sec. 3, the uniform FBE has a signal delay which is about half of that of the corresponding uniform AS FB for sample-wise processing. The same applies for the APT FBE compared to the corresponding APT AS FB. In [7] it has been shown that the proposed FBE can be used for noise reduction with reduced signal delay. Instrumental measures revealed that the achieved noise reduction by means of the FBE and the corresponding AS FB (with the same values for L, M and r) is approximately equal. For noise reduction, the filter coefficients are smoothed by the noise reduction algorithm to avoid so-called 'musical tones'. Such smoothing can also avoid filter-ringing effects which might occur for time-domain filters with strongly varying coefficients.

6. CONCLUSIONS

In this contribution an efficient filter(-bank) structure has been developed, which results in a time-domain filter based on coefficients adapted in the spectral-domain. The proposed generalized filter-bank equalizer accounts for a uniform and non-uniform frequency resolution by means of frequency warping, and a broad class of spectral transforms can be realized. (Near) perfect signal reconstruction can be achieved at lower effort than for a common analysis-synthesis filter-bank. The filter-bank equalizer has a lower signal delay and needs less delay elements (memory) than the corresponding analysis-synthesis filter-bank.

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